Psychoacoustical Dissonance as a Tool for Musical Analysis

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Abstract

The human auditory system's remarkable ability to perceive dissonance has prompted extensive research on the subject, including the work of Hermann von Helmholtz, R. Plomp & W. J. M. Levelt, and William Sethares. This article explores the complex nature of psychoacoustical dissonance, proposing a model to analyze chords and vertical sonorities in musical excerpts, considering the direct perception of music and its relationship to the human auditory system. Building on the foundation laid by previous research, this model offers a new perspective for examining the intricacies of musical excerpts, providing valuable guidance for nuanced performance, and potentially unveiling a different understanding of how humans perceive dissonance. By integrating this model with other analytical methods, a more comprehensive approach to musical analysis can be achieved, enriching our appreciation and experience of music as a whole. As our understanding of psychoacoustical dissonance and the human auditory system continues to evolve, so too will our approach to interpreting and performing music, ultimately enhancing our connection to the rich tapestry of sound that surrounds us.

1 Introduction

In 1965, R. Plomp and W. J. M. Levelt published a study which quantified the perception of dissonance and its direct relation to critical band theory[4]. The human auditory recognition system is the most highly developed of any living organism. Stemming from our prehistoric need for the immediate recognition and assessment of threats, humans are remarkably talented at deciphering sounds that are seemingly identical. In an article titled, "The Paradox of Timbre", Cornelia Fales remarks that "timbre is critical to human contact with the environment and a sonic dimension we track with peculiar sensitivity"[1]. Rebecca Leydon explains "Hearing is not simply a matter of unilateral sound pressure on the auditory nerve, but rather a cascade of intricate electrochemical events within the body."[3] German physicist Hermann von Helmholtz first posited that the human perception of dissonance is caused by the idea of roughness, or two sinewaves which frequencies become so close to each other that they create the phenomenon of beating (a periodic rhythm which is heard based on the phase cancellation of the two waves).[2] Plomp & Levelt take this theory and expand it by adding the realization that the frequency of the two adjacent tones greatly alters the perception of dissonance. For example, two pure tones at 1000hz and 1002hz (with a beating frequency of 2hz) will have a lower dissonance level than two pure tones at 200hz and 202hz. The reason for this phenomenon is because of critical bandwidth. Critical band theory states that while the human ear can discern similar sounds and tones, there exists a threshold in which the ear becomes "irritated" by adjacent frequencies blending together into a single perceptual "band". Plomp & Levelt summarize Hermann von Helmholtz's discoveries, stating that "for small frequency differences the beats between two simple tones can be heard individually, but for larger distances this becomes impossible, due to their rapid succession, and the sound obtains a rough and unpleasant character. He ascertained that this roughness has a maximum for a frequency difference of 30—10 cps, independent of frequency, but admitted also that for a constant difference the roughness increases with frequency. For larger frequency differences, roughness decreases and the sound becomes consonant and agreeable, independent of frequency ratio."[4] Ultimately, as both theories of Helmholtz and Plomp & Levelt involve interactions of individual pairs of pure tones, their resulting dissonance characteristics end up being comparatively quite similar. In their article, Plomp & Levelt propose a "Standard curve representing consonance of two simple tones as a function of frequency difference with critical bandwidth as a unit."[4]

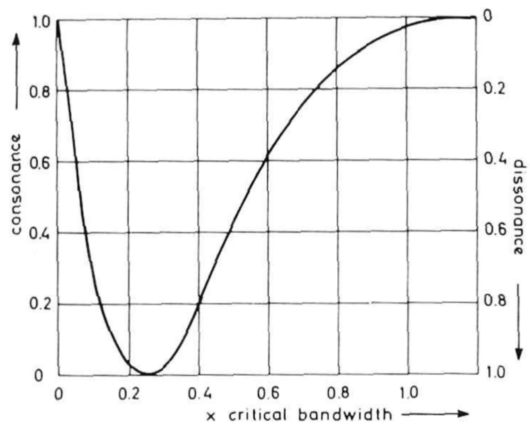

Figure 1: Critical bandwidth curve created for two sine waves at diverging frequencies. Figure created by Plomp & Levelt.[4]

In a later study (1993), William Sethares added to the standard curve by overlaying different base frequencies to show the change in critical bandwidth based on the location of the frequencies along the audio spectrum. Sethares' figure illustrates that lower frequencies have a larger band, meaning that two low frequencies can be further apart and still contain a high level of psychoacoustical dissonance, while higher frequencies must be closer to be perceived as dissonant. The studies of von Helmholtz, Plomp & Levelt, and

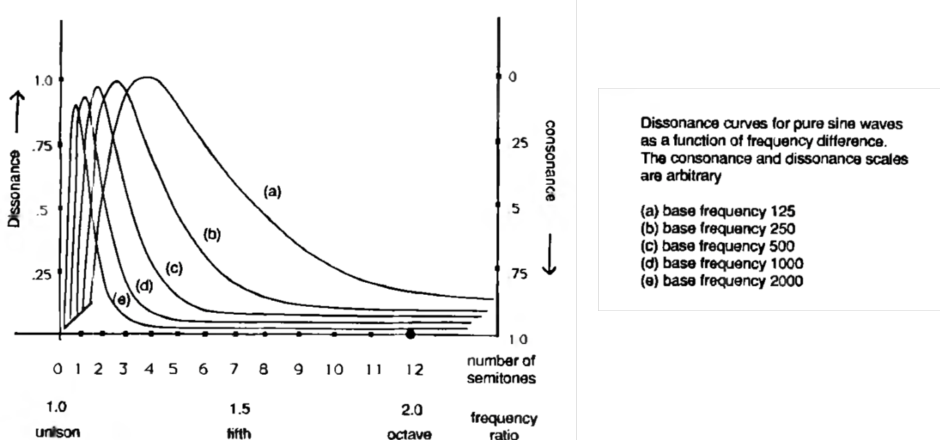

Figure 2: Sethares' addition to Plomp & Levelt's consonance curve, with a flipped Y-axis illustrating the difference in critical bandwidth across the audio spectrum.

Sethares have created a foundation for a new understanding of dissonance interpreted as a complex gradation. William Sethares' model expands on Plomp & Levelt's research by proposing a model for calculating the dissonance of complex tones by adding together every pair of pure tones present in a complex tone, and subsequently calculating the dissonance of two (or more) complex tones played simultaneously. This article applies both Plomp & Levelt and Sethares' research to propose an application for this model to derive meaningful analysis by calculating the psychoacoustical dissonance of chords and vertical sonorities of musical excerpts. These analyses can be vital in contemplating how certain musical excerpts should be performed by understanding how music interacts directly with the human ear in regards to the phenomenological perception of dissonance in time-based musical art.

2 Calculating the dissonance of a chord

After the 1965 study on psychoacoustical dissonance by R. Plomp and W. J. M. Levelt, William Sethares published an article in which he details a model for calculating the psychoacoustical dissonance of two complex tones expressed as one base tone, and another tone higher tone at a certain interval above the base tone[5]. This allows for a simple way to derive the dissonance curve proposed by Plomp & Levelt's[4]. We can expand Sethares' model for calculating the dissonance of any two identical complex tones so that we can then calculate the dissonance of any chord or vertical sonority in a piece of music. The following model calculates the dissonance of every pair of pure-tone harmonic partials in every pair of notes in a given chord. Equation 1 is a summation of these note pairs (or intervals) in order to derive a value for the dissonance of a given chord.

Let C be a chord consisting of n amount of c notes, where c_i represents the fundamental frequency of each note c in chord C. For example, a simple A4 major chord with $n = 3$ tones would have the following values: $c_1 = 440, c_2 \approx 554, c_3 \approx 660$, or $C = [440, 554, 660]$. We will use Sethares' proposed standard complex tone containing 1 fundamental pure tone and 6 pure tone harmonic partials. Each partial will decrease in amplitude at a rate of 88% of the previous amplitude. Because the partials are harmonic, for any given note in the chord, we can derive the frequency of each partial by multiplying the base frequency by the partial number. The frequency of the fourth partial in note c_3 is simply $4c_3$, and the corresponding amplitude would be 0.88^{4-1}. For each possible pair of notes c_j and c_k in chord C, we must calculate the dissonance D of the partials (l and m) from both notes when played together. Variable s determines the critical bandwidth of all pairs (j and k) of pure tones in both sets of harmonic partials found in the two notes c_j and c_k.

$C = [c_1, c_2, \ldots, c_n]$.

$$D(C) = \sum_{j=1}^{n} \sum_{k=1}^{n} D(c_j, c_k), \tag{1}$$

$$D(c_j, c_k) = \sum_{l=1}^{7} \sum_{m=1}^{7} 0.88^{(l+m)-2} \left(e^{-3.5sb} - e^{-5.75sb} \right), \tag{2}$$

$$s = \frac{0.24}{0.021 * \min(lc_k, mc_j) + 19}. \tag{3}$$

where $b = |lc_k - mc_j|$. If we wanted to use a different complex tone other than Sethares' standard, we could simply add more harmonic partials by raising the value of the final index of each summation inside the main parentheses. Similarly, we could alter the rate of amplitude decrease by changing the first decimal value after these summations from 0.88 to any number between 0 exclusive and 1 inclusive. It should also be noted that the numerical dissonance values are arbitrary, but are only useful when comparing results between different input values (or input chords). Because of this arbitrary nature, for the sake of readability of whole numbers, I have elected to multiply all of the calculated dissonance values by 100, and then I have rounded these multiplied values to the nearest whole number. The equation above does not reflect this multiplication. With our extension of Sethares' proposed model[5], we can now calculate the psychoacoustical dissonances of intervals, chords, progressions, excerpts, and entire compositions, which in return provides a framework for analysing how composers use psychoacoustical dissonance in a way that differs from the concept of binary tonal dissonance and post-tonal interval classifications, and can directly correlate to how the human ear perceives the concept of dissonance and consonance.

3 Understanding interval calculations

In many traditional systems of musical analysis, the concept of consonance and dissonance is that of a binary. Intervals are typically seen as either consonant or dissonant, and chords, when given the context of their musical surroundings, are often also seen as either consonant or dissonant. This duality is expressed in tonal music as a dissonant interval needing to resolve to a consonant interval. Ever since the beginning of the twentieth century, the concepts of tonality, and reliance on harmonic resolution of dissonance has waned as composers began creating music that defied the standard structures of tonality. The twentieth century also gave birth to Post-Tonal theory and analytical techniques, creating new ways to depict intervals by a numerical value representing the amount of semitones (Pitch-Class Interval). Traditional Post-Tonal theory also uses the concept of Interval Class, where all intervals that create the same compound interval are all given the same identifying number. For example, a minor second, major 7th, minor 9th, and major 15th are all classified under Interval Class 1. This theory provides a simple way of grouping like intervals, but loses the concept of individuality between intervals that are perceived to be different.

Interval Name	U	m2	M2	m3	M3	P4	TT	P5	m6	M6	m7	M7	P8
Binary Tonal Consonance	C	D	D	C	C	C	D	C	C	C	D	D	C
Pitch-Class Interval	0	1	2	3	4	5	6	7	8	9	10	11	12
Interval Class	0	1	2	3	4	5	6	5	4	3	2	1	0
Perceived dissonance (100)	0	100	75	60	54	41	53	23	47	38	43	49	5

Table 1: Comparison of Different Interval Descriptions

We can use the concept of psychoacoustical dissonance, and the dissonance curve calculated in Sethares'[5] article to propose a new approach to the concept of musical dissonance. For example, by calculating the dissonance of each interval from a minor second to a perfect octave with the base frequency being C4 (261.63Hz), and (for the sake of comparison with simple numbers) dividing the values so that the most dissonant interval equals 100, we derive a collection that gives every interval a unique value. Table 1 compares different systems for categorizing intervals within an octave. Each system gives a predictable data set, with linear relationships to adjacent intervals. The concept of psychoacoustical dissonance and critical bandwidth when used in this context creates a data set that is not linear, and relates directly to the perception of sound, rather than an easily describable ordering of numbers. Figure 3 compares the data from Table 1 for Interval Class and psychoacoustical dissonance (from 0 - 100).

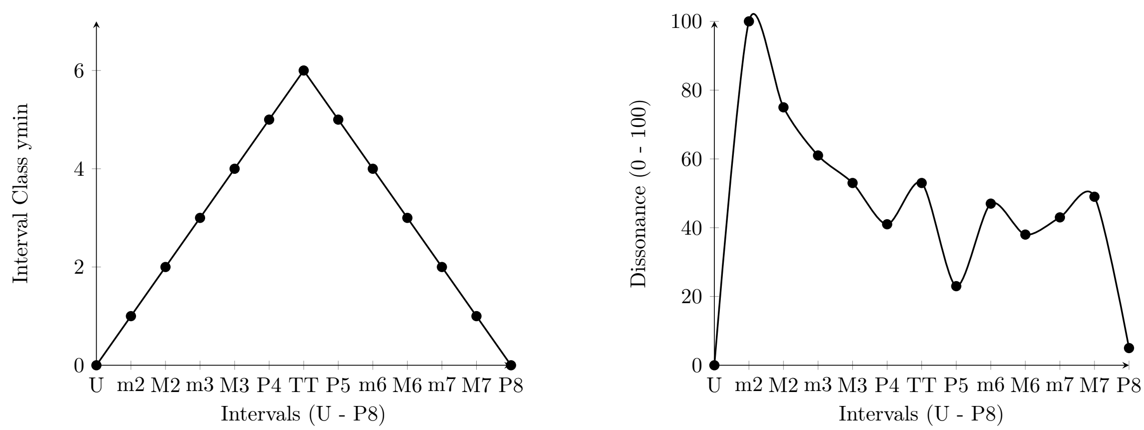

Figure 3: Comparison of Perceived Dissonance and Pitch-Class Interval as discrete values to represent intervals within one octave.

In Table 1, and in the second graph of Figure 3, the data for psychoacoustical dissonance correlates the four most dissonant intervals with the smallest intervals in order. The curious result of this data is that both the minor and major third are more dissonant than the tri-tone, which is a direct contradiction to most theories of harmony, where a dissonant tri-tone would typically resolve to a consonant third. With that being said, in order to use these calculations as a way to analyze dissonance in a larger scale, we must abandon the preconceived understanding of binary Tonal dissonance, as the alternative psychoacoustical dissonance posits that intervals can be placed on a gradient scale of dissonance, and the degree of dissonance will increase if the partials of the two tones are more likely to overlap within each other's critical bands. Using psychoacoustical dissonance to describe intervals illustrates a higher level of distinction between intervals that in other descriptive systems have the same values. Table 2 gives an example of intervals within two octaves that are all classified under the same Interval Class, but possess drastically different values from one another using psychoacoustical dissonance data.

In order to relate my analysis to a broader scope of harmony, I will continue to use the term *dissonance*, but unless specifically stated, I will be referring to psychoacoustical dissonance, rather than Tonal dissonance.

Interval Name	m2	M7	m2+8	M7+8
Interval Class	1	1	1	1
dissonance	435	214	196	86

Table 2: A comparison of Interval Class values with dissonance values

4 Analyzing the dissonance of common tonal chord progressions

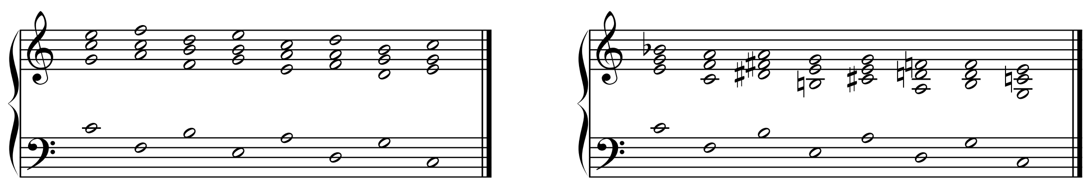

Figure 4: Two examples of common tonal harmonic progressions

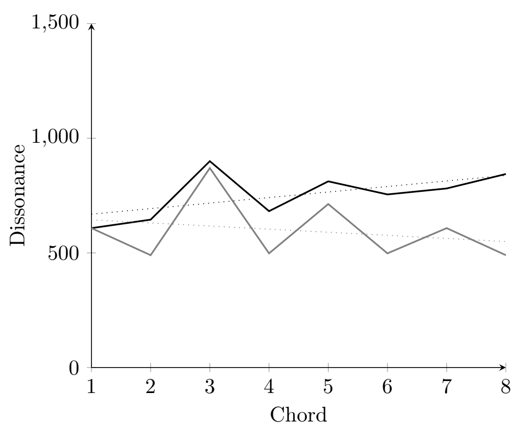

Figure 5: Average Dissonance: **753**

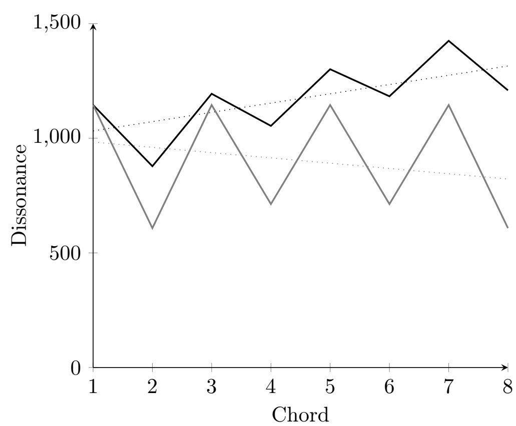

Figure 6: Average Dissonance: **1174**

Figure 7: Two common tonal chord progressions in C major: A Descending Fifths sequence, and a Secondary Dominant sequence

For the first application of dissonance as an analytical tool, I have used two common, tonal chord progressions. The resulting graphs show the dissonance of each chord in succession. These graphs - along with every other graph for the rest of the article - are formatted to show the harmonic dissonance (or true dissonance) value as the black solid line, and the transposed dissonance value as the grey solid line.

Both harmonic dissonance and transposed dissonance show their linear regression trend lines as dotted lines of corresponding shades. Interesting results are uncovered when juxtaposing a progression of dissonances against the same progression with all chords transposed to the same fundamental. By doing this, we can compare the resultant dissonance of the chord as it sounds in context of the excerpt against the dissonance value of each harmony when register is taken out of the calculation. In Figure 6, we see the harmonic dissonance line slowly increasing as the chords descend, while the high values of the transposed dissonance line remain near 1100. This is because the dominant-seventh sonorities of the progression are all voiced identically, giving the same transposed dissonance value, but different harmonic dissonance Values as the progression descends. We see a difference in transposed dissonance between Chord 2 and Chord 4 & 6. This distinction highlights the difference in dissonance between a major and a minor chord with the exact same voicing. The distinction here is not as obvious when focusing on the harmonic dissonance line. This is one of the reasons for including both lines. We will see in later analyses the interesting relationship between harmonic dissonance and transposed dissonance, as well as the role of the linear regressions.

5 Alexander Scriabin - Piano Sonata No. 4

Figure 8: Scriabin Sonata No. 4 (mm 1 - 8) with numbers indicating each vertical harmony (illustrated in Figure 9)

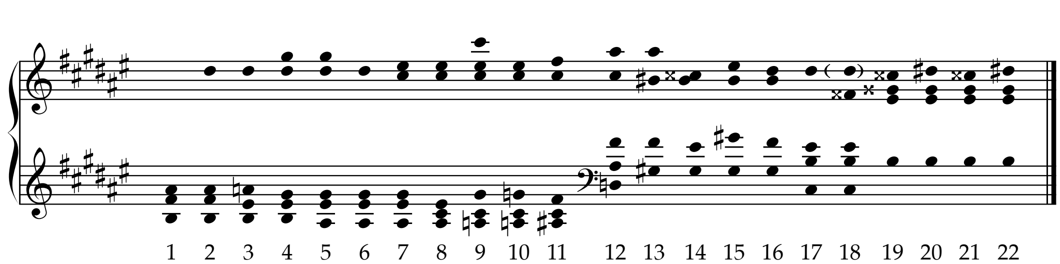

Figure 9: Sounding vertical sonorities in Scriabin's Sonata No. 4 (mm 1 - 8)

Russian composer Alexander Scriabin lived during a transitional time in Western Music History. With the majority of his mature output being composed between the years of 1892 and 1915, the development of Scriabin's harmonic language can be charted through analyzing his 10 Piano Sonatas. When listening to all ten sonatas chronologically, one can hear a slow departure from common practice idioms, and a development of Scriabin's deeply personal harmonic language. There are many points in which Scriabin explores gestures that are quite similar to the those developed in previous sonatas. We will focus on a few of these excerpts, where Scriabin's interest in carefully selected, contrapuntal harmonies are brought to the fore. In these excerpts, Scriabin often articulates sonorities through time, using contrapuntal techniques and independent articulation of harmonic voices. Because of the percussive nature of the piano, the ear is drawn to each new attack from the myriad of harmonic voices sounding independently. As each voice moves, whether on its own, or with another voice, the color of the ringing harmony changes. Scriabin's persistent interest in these harmonic moments leads me to believe that there could be a relation between his choice of harmonic movement and the psychoacoustical dissonance of these harmonies. Many articles have been written explaining Scriabin's interest in color, as he was a known synesthete. I believe that it is highly probable that Scriabin had a predilection for what he might have thought of as the color of a harmony, but could also be interpreted as his perception of dissonance in a psychoacoustic context.

In the first eight measures of Scriabin's Piano Sonata No. 4, we can observe a harmonic ambiguity that is characteristic of his later works. The mixture of elaborated and non-functional major and minor chords creates a sense of instability and uncertainty. When analyzing this passage, it can be difficult to separate our preconceived notions of tonal hierarchy from the music itself. However, if we approach this excerpt with psychoacoustical dissonance model, we can analyse how the chords present themselves on a gradient of dissonance and consonance, potentially unlocking a new analytical perspective that can inform our performance of this and similar excerpts in Scriabin's music.

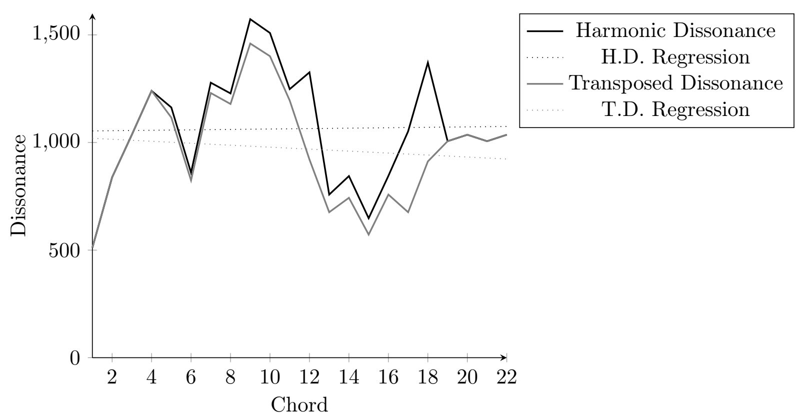

Figure 10: Scriabin Sonata No. 4, mm. 1 - 8
Average Dissonance: **1066**

From analyzing Figure 10, we can find a few interesting moments relating to the harmonic dissonance of this excerpt. The harmony from chords 1-3 grows in dissonance, reaching a local peak at chord 4. After this peak, The melodic leap downward to D♯5 lowers the dissonance considerably. As the harmony and dissonance builds again in preparation for the climax of the first phrase, chord 9, the harmony immediately preceding this chord shows a slight drop in dissonance. The inner voice's drop to a C♯4 briefly lowers the dissonance, making the dissonant arrival on chord 9 more salient. Scriabin also chooses to attack all of these notes at once as a block chord. This choice could reflect Scriabin's understanding of psychoacoustical dissonance, as this chord is the dissonance peak of the first musical sentence.

When looking closer at chords 11, 12, and 13, we notice the transposed dissonance value of chord 12 diverges from that of the harmonic dissonance. This is reflected in the voicing that Scriabin chooses. Chord 12 is a widely spaced, augmented seventh sonority with a fundamental of D3. From the transposed dissonance line, we see that chord 11 12 and 13 create a straight, downward slope, but the harmonic dissonance line differs from this slope, with chord 12 usurping the dissonance of chord 11. As we have seen from analyzing the previous tonal progressions, when charting dissonance in this fashion, there is a bias towards chords that have lower frequencies, as lower frequencies have a larger critical bandwidth, allowing for more low intervals to have higher dissonance. Although this chord is voiced so that it has the lowest fundamental thus far, chord 9 still has a higher dissonance value. Even though chord 12 provides a registral shift downward, the focal point still remains on chord 9, meaning that the character of chord 12 might be played in submission to that of chord 9.

Further performance justifications can be made by analyzing the dissonance of chords 17 and 18. There is a local dissonance peak is at chord 17, though this peak does not usurp that of chord 9. In the manuscript, the D♯5 on the downbeat (chord 17) is held to the length of a quarter note, bleeding over to the following sonority (chord 18) which includes an F𝄪4 (In Figure 9, this is shown by a parenthetical D♯5 in chord 18). From looking at the music using a more traditional analytical framework, we get a sense of ambiguity as to whether the F𝄪4 is a member of the preceding melodic line, or if its role is to be an inner-harmonic color shift. This ambiguity incites the performance question of whether to truly hold the D♯5 over to bleed into the sonority of the F𝄪4, or to allow the F𝄪4 to sound as the highest note in the sonority for chord 18. When juxtaposing these two realizations on our dissonance graph, we uncover an interesting distinction.

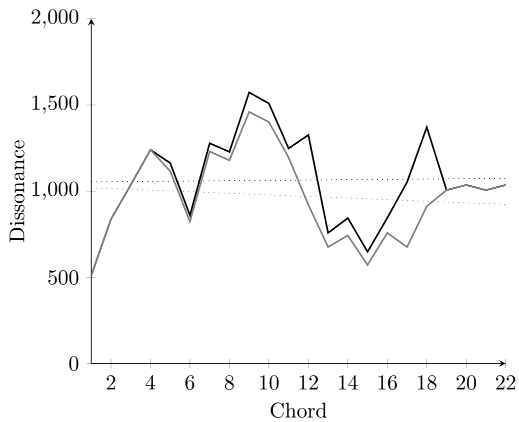

Figure 11: Sonata No. 4, mm. 1 - 8 **without** D#5 sounding in mm. 7
Average Dissonance: **1066**

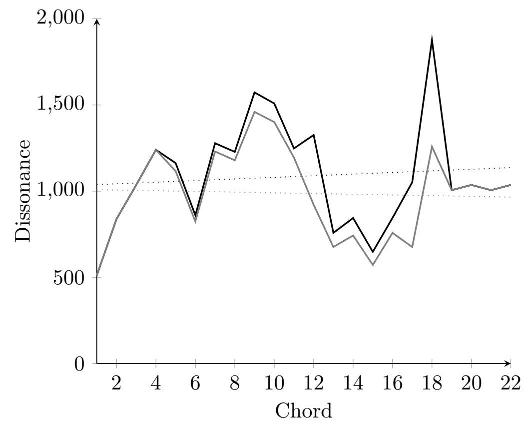

Figure 12: Sonata No. 4, mm. 1 - 8 **with** D#5 sounding in mm. 7
Average Dissonance: **1094**

Figure 11 and Figure 12 illustrate a major change in both local dissonance and the global dissonance of the first full sentence when allowing the D♯5 of chord 17 to ring into the following chord 18. Because of this, Figure 12 contains a new dissonance peak at chord 18. This peak is much higher than that of chord 9. This change, in return alters the topography of the entire passage. In both figures, we see three distinct peaks, at chord 4, chord 9, and chord 17 or 18. To categorize the trajectory of only the peaks of the graphs, we see that in Figure 11, chord 4 is the lowest peak, then chord 9 is the highest, and chord 17 is in between the two, creating a low-high-middle, or even an arc-like relationship. In contrast, Figure 12's peaks rise linearly in a low-middle-high relationship. By analyzing these differences in psychoacoustical dissonance, we find a new parameter in which performers can treat with subtlety in order to craft different arcs and tension points within phrases.

Finally, we see the harmonic and transposed dissonance converge at chord 19, giving identical values up to the end of the excerpt. As we have seen from the tonal progressions, this convergence does not always happen, but one could deduce that both the beginning and end of the musical sentence being harmonized so that the harmonic dissonance is exactly the same as the transposed dissonance gives a sense of closure, completeness, and stability, as the harmonies are in their "correct" position in relation to the beginning of the excerpt.

6 Alexander Scriabin - Piano Sonata No. 5

The introductory material in Scriabin's Piano Sonata No. 5 is an exercise in juxtaposition of musical characters. After a brief, fiery flourish of an introduction, the music jumps to an extremely introverted state. This phrase, from measure 13 to 24, seems to reflect the often highly distant, contemplative mood found in much of Scriabin's music. The power of this character is only heightened by the previously brash

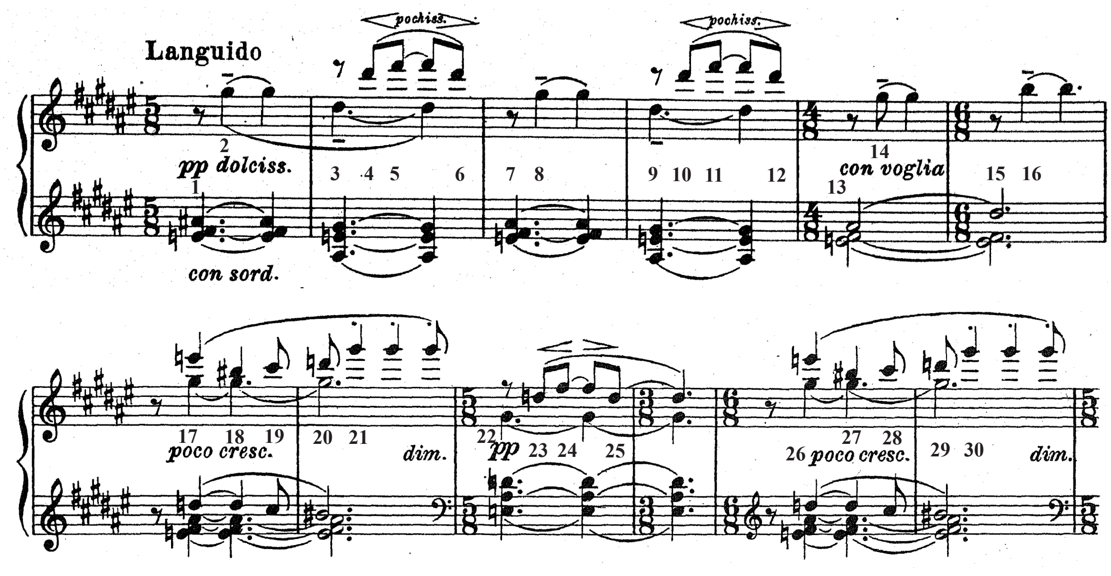

Figure 13: Scriabin Sonata No. 5 (mm 13 - 24) with numbers indicating each vertical harmony (illustrated in Figure 14)

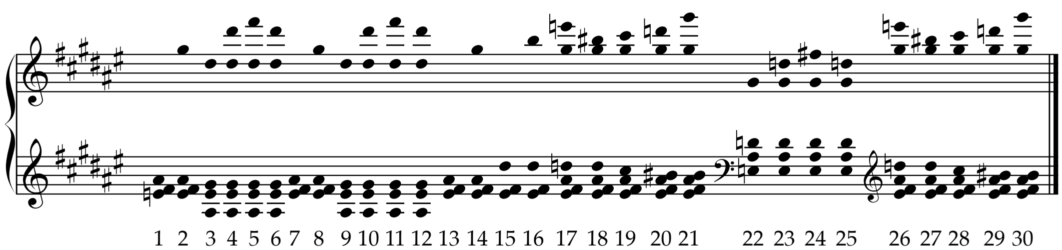

Figure 14: Sounding vertical sonorities in Scriabin's Sonata No. 5 (mm 13 - 24)

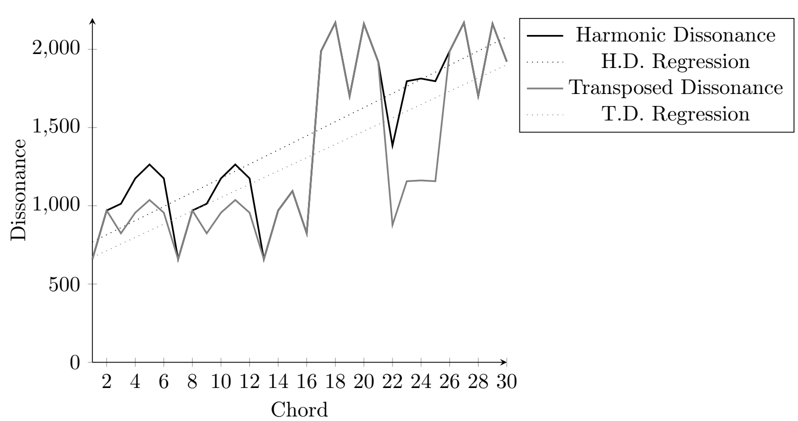

Figure 15: Scriabin Sonata No. 5 (mm. 13 - 24)
Average Dissonance: **1425**

introduction. At a more local point of view, the music in this introverted section, marked *Languido*, creates another duality between measures 13-18, and measures 19-24 (chords 1-16 and 17-30 in Figure 14). This duality, while recognizable from a framework of motivic analysis, is also very clearly depicted in our dissonance analysis in Figure 15. We see two main fields of dissonance, from chords 1-16 (measures 13-18) occupying around 600 to 1400 harmonic dissonance units, to chords 17-30 (measures 19-24) occupying a much higher dissonance field of 1400 to 2200 harmonic dissonance units. We can also see a high level of gestural repetition from this section, where chords 1-6 are identical to chords 7-13. Likewise, chords 17-21 are identical to chords 26-30. There is a remarkable separation of harmonic and transposed dissonance values from chords 22-25. The transposed dissonance is much lower at this moment, and even within the field of the first section from chords 1-16, while the harmonic dissonance is much higher, and similar to that of its surrounding repeated phrases of chords 17-21 and 26-30. Analysing the contour of the left hand in this section also shows a motivic similarity to that of chords 3-6. This special moment shows that Scriabin is taking the duality between chords 1-16 and 17-30 and momentarily fusing their characteristics together in chords 22-25 by producing a motivic variation of the first section, but transposed lower so that it gives the same local dissonance values of the surrounding material in the second section. This creates a brief shadow of the first section while simultaneously remaining grounded in the sonic dissonance of the second section. This realization might not be so easily deducible without analysing the harmonic and transposed dissonances of the excerpt.

The largest discrete change in dissonance exists from chord 16 (around 750 units) to chord 17 (around 2000 units). This leap is highlighted by the lowering of dissonance between chord 15 and 16. This gesture of lowering the dissonance level just before leaping to a very high dissonance is also used in the previous except in Sonata No. 4, and the relationship between chord 7, 8, and 9. Chord 8 lowers the dissonance from chord 7, and then leaps to a high dissonance in chord 9. The lowering of dissonance just before leaping to a high dissonance creates an even higher perception of the contrast in dissonance between the locally low and high-valued chords. In a more traditional lexicon, preceding a dissonant chord with a much more consonant chord will result in the dissonant chord sounding *more* dissonant than if it was preceded by a similarly dissonant chord.

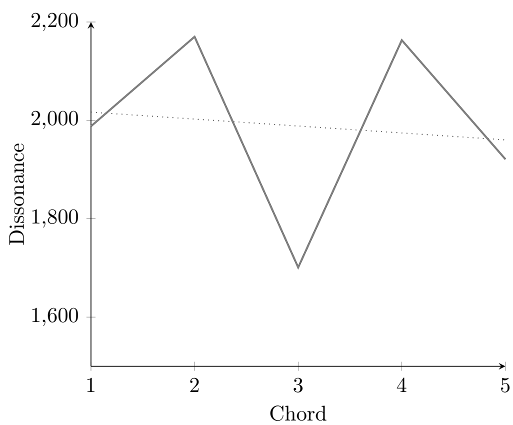

Figure 16: Scriabin Sonata No. 5, chords 17 - 21 (or 26 - 30)

There is a local consonance (low dissonance) in the progression between chords 17 through 21 (and 26 through 30). Chord 19 possesses a much lower dissonance value because of the voice exchange present between the two contrapuntal lines B♯5→C♯6→D6, and D5→C♯5→B♯4. When both contrapuntal voices play the C♯ simultaneously, the resulting chord is much less dissonant than the adjacent sonorities in this voice exchange. Figure 16 illustrates chords 17-21 (or 26-30) as a closeup, where the graph forms a familiar wavelike relationship with a slight downward tilt. In contour notation, this could be described as <24031>. This recognizable relationship is one of value, and can inform both the musical analyst and the performer of the unique quality of this musical moment. Identical to the previous excerpt of Scriabin's Sonata No. 4, this excerpt also displays a convergence of Transposed and harmonic dissonance at the end of the musical sentence, conveying similar feelings of closure and completeness.

7 Comparing Sonata No. 4 to Sonata No. 5

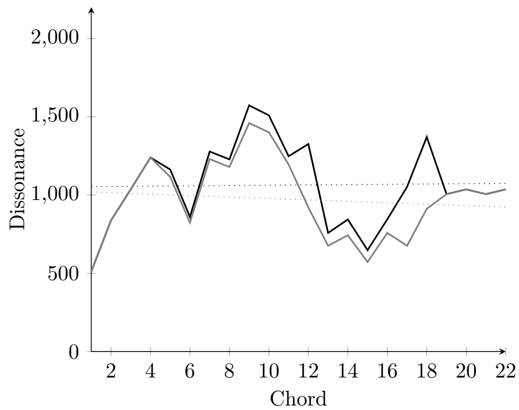

Figure 17: Scriabin Sonata No. 4, mm. 1 - 8
Average Dissonance: **1066**

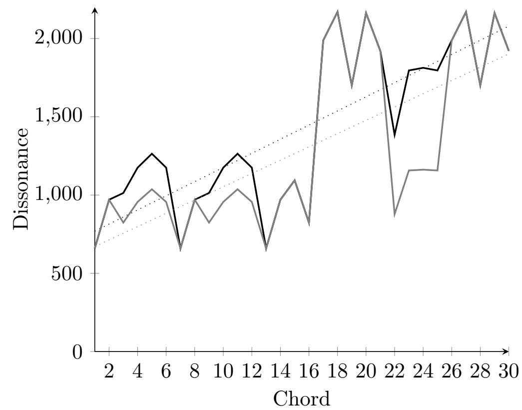

Figure 18: Scriabin Sonata No. 5, mm. 13 - 24
Average Dissonance: **1425**

By placing the dissonance graphs of the excerpts from Sonata No. 4 and Sonata No. 5 next to one another, we are able to analyse the similarities and differences between the two. We see that there is a major difference between the linear regressions (dotted lines) from each excerpt. The trend line in Sonata No. 4 excerpt is perfectly flat. This contrasts the steep upward slope of the Sonata No. 5 excerpt. The harmonic dissonance regression spans from around 750 dissonance units to over 2000 by the end of the excerpt. This is because of our previously discussed dissonance range dualities existent in this excerpt in which the first half is considerably less dissonant than the second half. This duality does not exist in the Sonata No. 4 excerpt and is reflected by the regression remaining flat as the harmonic dissonance following a pseudo-cyclical trend. Below each graph is an Average dissonance value, used to compare the harmonic dissonance mean of multiple excerpts. We see a higher Average dissonance value in the Sonata No. 5 excerpt, which can be aurally perceived, and may even relate to the tonal characteristics of these excerpts in which the Sonata No. 4 excerpt loosely conveys a tonic progression in F♯or C♯major, while the Sonata No. 5 excerpt is much more tonally ambiguous. Sonata No. 5 is also the final sonata in which Scriabin chooses to use key signatures.

Another key difference between these excerpts is the use of repetition at a local level. In Sonata No. 4, Scriabin uses repetition, but at a higher level than that of Sonata No. 5. It is clear in Sonata No. 5 that Scriabin is using short repetitions to emphasize the dualities mentioned earlier. This repetition in return produces different contours on the dissonance graphs. Figure 19 illustrates these contours more clearly. As

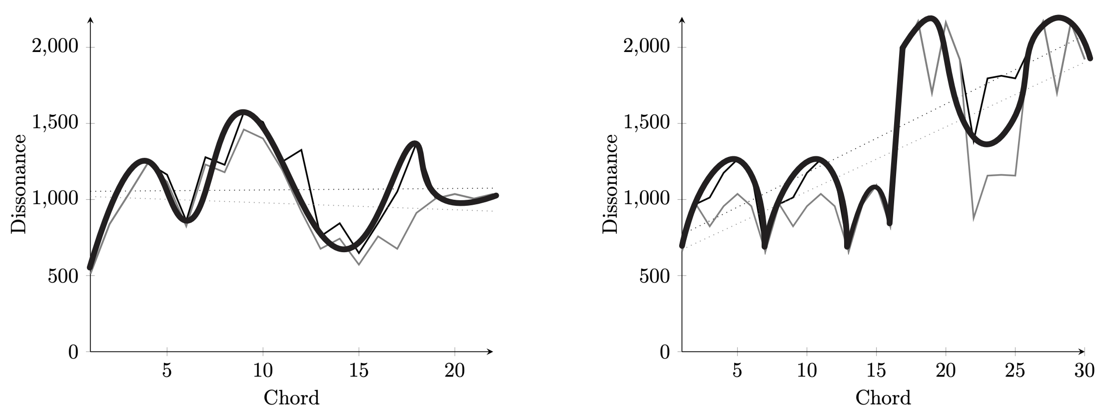

Figure 19: Visualization of the general contour created by each the dissonance in each excerpt.

we see in Figure 19, the Sonata No. 4 excerpt's contour is much more gradual and continuous, creating a smoother line. By contrast, the excerpt from Sonata No. 5 creates a contour line that has sharper low points (chords 7, 14, and 16). This aurally presents as a more gripping, sudden change in dissonance, rather than each high and low point being validated by its surrounding sonorities' similar dissonance values (as seen in Sonata No. 4). We also see the semi-sinusoidal contour of chords 17-30 in the Sonata No. 5 excerpt. This is a larger scale sinusoid than encompasses both smaller sinusoids from Figure 16.

8 Other examples of psychoacoustical dissonance as an Analytical Tool

The psychoacoustical dissonance model can be used to analyse and compare all types of harmonies in music. The best musical applications for this model are excerpts that mostly contain chords that are articulated at once or nearly simultaneously. One can use this model to analyse harmonies that are played in a more contrapuntal style (similar to the Sonata No. 4 excerpt), but in order to truly understand the perception of dissonance in such a passage, one must take into account the composer's choice to articulate the sonorities through time, and not at once. A potentially interesting application of this model could be to analyse the chorales of J.S. Bach. Most of these chorales involve continuous homorhythmic harmonies with passing tones that alter the psychoacoustic dissonance based on their function as non-chord-tones within a tonal context. I have chosen to use the beginning of Bach's chorale setting of *Aus meines Herzens Grunde* (BWV 269). In a contrasting example of a twentieth-century excerpt which also uses primarily homorhythmic harmonies is the opening of the second act in Igor Stravinsky's *Le Sacre du Printemps*.

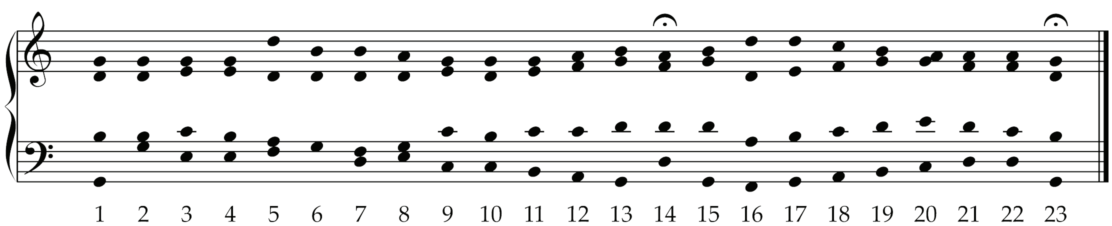

Figure 20: J.S. Bach - *Aus meines Herzens Grunde* (BWV 269) first two phrases.

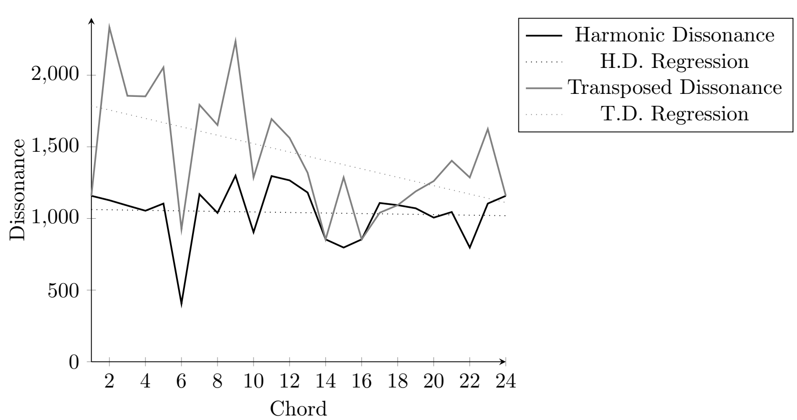

Figure 21: J.S. Bach - BWV 269 (first two phrases)
Average Dissonance: **1041**

In the Bach excerpt, an interesting relationship between harmonic and transposed dissonance arises. In the model, the first chord sets the transposed dissonance to be equal to the harmonic dissonance. In the second chord of the chorale, the only change is that the bass rises up an octave. In the graph, the transposed

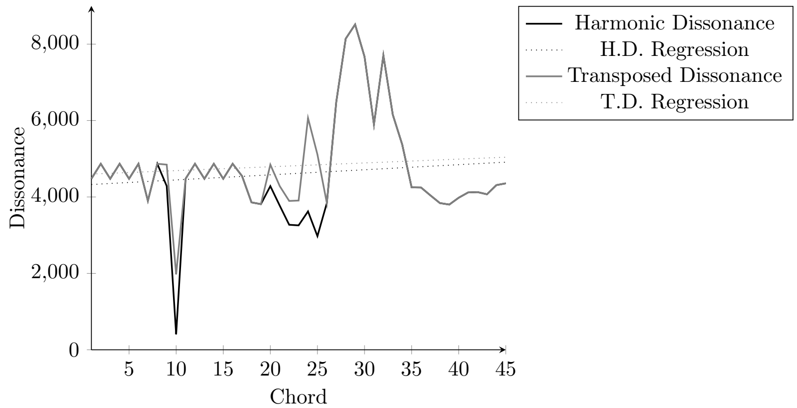

Figure 22: Igor Stravinsky - *Le Sacre du Printemps* - Act II Opening
Average Dissonance: **4619**

dissonance leaps to a very high level because of the second chord being read as an octave lower than written (counteracting the bass's upward leap). The Transposed and harmonic dissonances remain quite separate until they reach a point of convergence around chord 14. They then separate again after this cadence. This creates an interesting, convergence-then-divergence relationship between the harmonic and transposed dissonances.

In the Stravinsky excerpt, we see that the Transposed and harmonic dissonance lines are quite similar throughout the entire excerpt. We also notice a very large downward leap at chord 10. This is quite evident when looking at the score for this excerpt. Stravinsky lightens the chordal texture here, almost as a harmonic breath mark. Of course, many more analytical remarks can be made about these two excerpts, but for the sake of brevity, I am only proposing these excerpts as potentially interesting examples for further analysis.

9 Possible challenges of the psychoacoustical dissonance analytical model

Using the analytical model of charting the psychoacoustical dissonance of chords in a musical excerpt uncovers noteworthy results that can be applied both to the understanding of a composer's intent, and to the subtly in performance of said excerpts. In order to apply this model to a broad range of excerpts, a few constants were used that make the calculations more straight forward, but in return make the contrived dissonance values imperfect. The standard timbre[5] proposed by Sethares of 7 partials that decrease in amplitude by 88% of the previous partial is a timbre that loosely reflects a "regular complex tone", but lacks the imperfection of an exact model of, for example, a piano's timbre. The proposed model can easily include a variable for both the amount of partials p, and the rate of amplitude decrease q. Equation 5 reflects this change. If one wanted to use a more complex timbre with other parameters, or possibly an inharmonic (without perfect-multiple partials) timbre, one would likely need to expand the model to account for these parameters. The concept of inharmonic timbres and their resulting dissonance is part of the research done by Sethares in *Local consonance and the relationship between timbre and scale*[5].

Another parameter that the model purposefully excludes is the amplitude of each tone in a chord. Oftentimes, and even in the Scriabin excerpts, chords are not played with every chord tone being attacked at the same time. Many instruments, including piano, have amplitude envelopes that follow a fast onset, and slow release pattern, where the highest amplitude a given note will possess will happen during the attack stage of the note, and the longer the note rings, it will decrease in amplitude continuously. Composers exploit this characteristic, exactly how Scriabin did in the opening of his Sonata No. 4, by attacking notes at different times, which in turn shifts the the psychoacoustical dissonance of the sounding chords. A simple example would show that a C major chord with 3 notes would have a higher dissonance value if all notes were struck together, versus if each note was played as an arpeggio where all of the notes rings over each other as they lose amplitude. Adding this calculation to the model would complicate both the equation and the derived graphs, as it becomes harder to directly relate the salient elements in the graphs to other parameters that are commonly analysed, like register and spacing. One way to alter the model is to have it take a set of ordered pairs, each with a frequency c_i (like before), and a corresponding amplitude value a_i, where $0 < a_i \leq 1$. The following equations illustrate this change, including our parameterization of the timbre's partials and corresponding amplitudes.

$$C = (c_1, a_1), (c_2, a_2), \ldots, (c_n, a_n). \tag{4}$$

$$D(c_j, c_k, a_j, a_k) = a_j a_k \sum_{l=1}^{p} \sum_{m=1}^{p} q^{(l+m)-2} \left(e^{-3.5s|lc_k - mc_j|} - e^{-5.75s|lc_k - mc_j|} \right) \tag{5}$$

As stated above, these additions only complicate the equation and the graphs that they create. The original proposed model works as a means to discover a new perspective on certain aspects of chordal excerpts in music. Because of the model's use of critical bandwidth and psychoacoustical dissonance, the salience of these aspects is directly tied to how we hear music. The model should not be used to analyse music in a vacuum. It must be used in addition to other analytical methods. The model may also serve as a way to validate other analytical hypotheses or performance decisions on a basis of the direct perception of the music.

10 Conclusion

Over the past century, the research conducted by Hermann von Helmholtz, R. Plomp & W. J. M. Levelt, and William Sethares has advanced our understanding of psychoacoustical dissonance, unveiling its complex and dynamic nature, and laying a foundation for the exploration of perceptual nuances of dissonance with respect to the human auditory system. Building upon this research, the proposed model for analyzing music by means of charting the psychoacoustical dissonance in chords and vertical sonorities creates a new perspective for examining the intricacies of musical excerpts, and potentially unveiling a new understanding of how humans perceive dissonance in a different way than our previous notions of the subject. This approach allows us to delve deeper into the composer's intent and offers valuable guidance for the nuanced performance of these excerpts. The model's foundation in the human auditory system's capabilities and limitations provides a unique perspective on the phenomenological perception of dissonance in musical art, enriching our appreciation and experience of music as a whole. This model is intended to complement existing analytical techniques, enabling a more comprehensive understanding of musical analysis. By integrating this model with other analytical methods, we can achieve a well-rounded approach to musical analysis, providing new insights into the relationships between dissonance, harmony, and the auditory system. The model can also be used as a means to validate or challenge existing analytical conclusions or performance decisions, based on the direct perception of the music. This additional layer of analysis can lead to a more informed and intentional approach to performance.

As our knowledge of psychoacoustical dissonance and the human auditory system continues to evolve, our approach to interpreting and performing music will evolve to include such knowledge. The model proposed in this article serves as a means to advance our understanding of the relationship between musical analysis and musical perception. As we continue to understand this relationship further, we will in return enrich our experience and appreciation of music, and expand our ability to communicate through sound. Ultimately, the study of psychoacoustical dissonance and its relationship to the human auditory system presents an intriguing and promising area of research that has the potential to fundamentally alter our understanding of music, opening up new perspectives on both analysis and performance. By analyzing music through the lens of musical perception and psychoacoustical dissonance, we can continue to push the boundaries of musical expression, fostering a deeper and more profound connection to the rich tapestry of sound that surrounds us.

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